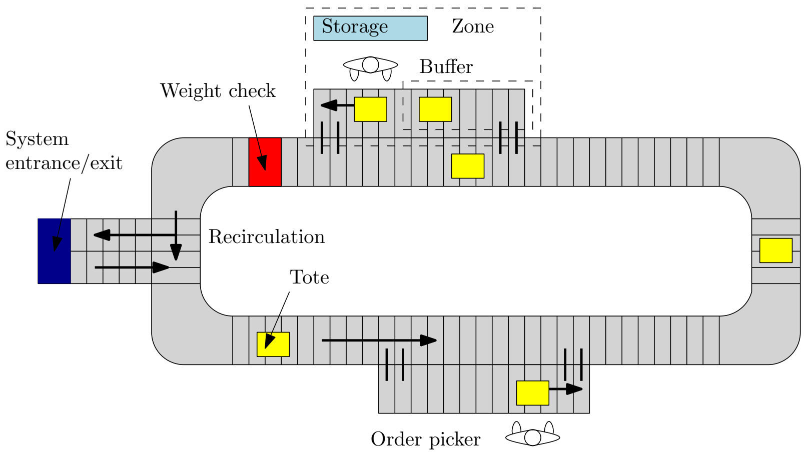
Exercise 8 Stochastic Models of Manufacturing Systems 4T400, 16 June

1. In a zone picking system with single-segment routing, three different elements can be distinguished: the entrance/exit of the system, the conveyor, and the zones. These elements of a single-segment zone picking system with two zones are shown in the figure below, where we can distinguish one entrance/exit, two zones, labeled zone 1 and 2, and three conveyor segments (connecting the entrance/exit and the zones).



At the entrance of the system, a customer order is assigned to a new order tote. The tote is released into the system as soon as it is allowed by the workload control mechanism. This mechanism keeps the number of totes in the system xed over time (say K totes) and only releases a new tote when a tote with all required order lines leaves the system. This workload control mechanism prevents the conveyor to become the bottleneck of the system. After release, a tote moves to the buffer of a requested zone (which is assumed to have ample space). When the picking process has finished in a zone, the picker pushes the tote back on the conveyor. The waiting time for a suciently large space on the conveyor is considered to be negligible. The conveyor then transports the tote to the next zone to be visited (if any). A weight check at the end of the conveyor loop ensures that the tote contains all the required order lines. When the tote has visited all the required zones, it leaves the system at the exit and a new order tote is immediately released into the system. We assume the new totes are released one-by-one at an exponential rate μ_e. A fraction p_1 of the new totes only has to visit zone 1, a fraction p_2 only needs to visit zone 2, and the rest needs to visit both zones. The service times at zone i are exponential with rate μ_i. Each zone has a single order-picker. The travel time on a conveyor segment is $1/\mu_c$ seconds. We will assume that $1/\mu_e = 5$ seconds, $1/\mu_c = 100$ seconds, $1/\mu_i = 18$ seconds, and the release probabilities are $p_1 = \frac{1}{2}$, $p_2 = \frac{1}{4}$.

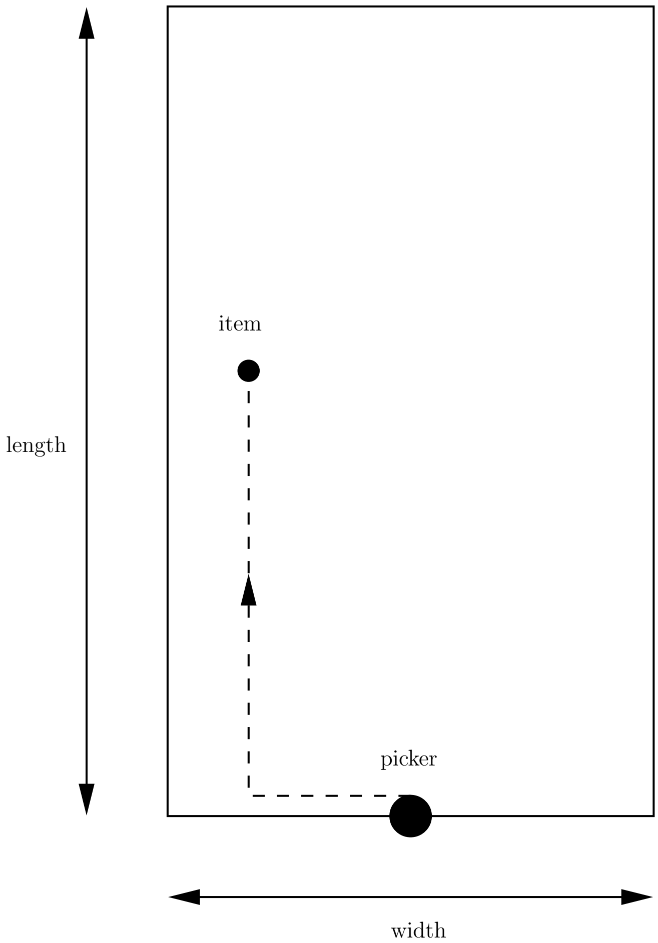
(a) Model this single segment as a closed queueing network.

(b) What is the bottleneck station? So what is the throughput as K tends to infinity?

(c) Formulate a mean value algorithm to compute the throughput, mean flow time, mean buffer contents at the zones, and the utilization of the order prickers, for workload threshold $K = 10, 50, 100$.

(d) By re-allocating the items to the zones, it may be possible to obtain more load balance between the two zones. Investigate the effect of load balancing on the performance of the system.

(e) The assumption of exponential picking times is a rough approximation. We now formulate a more accurate model for the picking times. The zone or storage area for a picker is shown in the figure below. The width of the storage area is 24 meter, the length is 36 meter, and the walking speed of the picker is 3 meter per second. The order totes are waiting in the buffer, located at the starting point of the picker. Assume that the item to be picked is randomly located in the storage area. The time to pick the item is 2 seconds. Compute the mean and standard deviation of the total picking time (walking time plus time to pick the item).



(f) Based on the above picking time model, adapt the mean value algorithm and compute again the performance.

(g) Suppose the storage area can be re-designed. While keeping the area of square meters the same, what are the optimal dimensions (in view of the system performance) and why?

Answer:

(a) The segment can be modeled as a closed network with K circulating totes and 7 stations: entrance/exit e, two zones z_1 and z_2, and three pieces of conveyor c_1, c_2 and c_3. The stations e, z_1 and z_2 have a single exponential server, with rate μ_e, μ_1 and μ_2 respectively. The conveyor pieces are modeled as infinite (or ample) servers with constant service times $1/\mu_c$. The visit ratios are denoted by v_e, v_1, v_2 and v_c, and these ratios are $v_e = v_c = 1$ and $v_1 = p_1 + (1 - p_1 - p_2) = 1 - p_2$, $v_2 = 1 - p_1$.

(b) The "relative" utilization of the entrance/exit e is $v_e/\mu_e = 5$, the relative utilizations of z_1 and z_2 are $13\frac{1}{2}$ and 9. Hence, zone z_1 is the bottleneck. As K tends to infinity, then the throughput of zone z_1 converges to μ_1 totes per second, and thus the throughput of the system becomes $\mu_1 v_e/v_i = 2/27 = 0.074$ totes per second.

(c) For $k = 1, \ldots, K$ totes, we have

$$\begin{aligned}
E(S_e(k)) &= E(L_e(k-1))\frac{1}{\mu_e} + \frac{1}{\mu_e}, \\
E(S_c(k)) &= \frac{1}{\mu_c}, \\
E(S_i(k)) &= E(L_i(k-1))\frac{1}{\mu_i} + \frac{1}{\mu_i}, \quad i = 1, 2, \\
\Lambda_e(k) &= \frac{kv_e}{v_e E(S_e(k)) + 3v_c E(S_c(k)) + v_1 E(S_1(k)) + v_2 E(S_2(k))}, \\
\Lambda_c(k) &= \frac{v_c}{v_e}\,\Lambda_e(k), \\
\Lambda_i(k) &= \frac{v_i}{v_e}\,\Lambda_e(k), \quad i = 1, 2, \\
E(L_e(k)) &= \Lambda_e(k)E(S_e(k)), \\
E(L_c(k)) &= \Lambda_c(k)E(S_c(k)), \\
E(L_i(k)) &= \Lambda_i(k)E(S_i(k)), \quad i = 1, 2,
\end{aligned}$$

starting with $E(L_e(0)) = E(L_c(0)) = E(L_1(0)) = E(L_2(0)) = 0$. The results are listed below.

K	$\Lambda(K)$	$E(S_e(K))$	$E(S_1(K))$	$E(S_2(K))$
10	0.030	5.76	27.4	23.5
20	0.055	6.76	53.0	33.3
50	0.074	7.94	453	54.0
100	0.074	7.94	1353	54.0

Clearly, for $K = 50$ station z_1 is already the bottleneck (so it does not make sense to add more totes).

(d) Suppose that by reallocating items, we can achieve $p_1 = p_2 = \frac{3}{8}$, so $v_1 = v_2 = \frac{5}{8}$. Then we get the following results.

K	$\Lambda(K)$	$E(S_e(K))$	$E(S_1(K))$	$E(S_2(K))$
10	0.030	5.76	25.3	25.3
20	0.056	6.78	41.6	41.6
50	0.085	8.68	223	223
100	0.088	8.90	665	665

Hence, the effect of load balancing is an increase of 20% in achievable throughput.

(e) Let X and Y denote the horizontal and vertical distance to the item, so $X = U(0, 12)$ and $Y = U(0, 36)$, with $E(X) = 6$, $\mathrm{var}(X) = \frac{1}{12} \cdot 12^2 = 12$, $E(Y) = 18$ and $\mathrm{var}(Y) = 108$. The total picking time P (walking plus picking) is equal to

$$P = \frac{2}{3}(X + Y) + 2,$$

so

$$E(P) = \frac{2}{3}(E(X) + E(Y)) + 2 = 18 \text{ (sec)},$$

and

$$\mathrm{var}(P) = \frac{4}{9}(\mathrm{var}(X) + \mathrm{var}(Y)) = \frac{160}{3} = 53.3 \text{ (sec}^2\text{)}, \quad \sigma(P) = 7.3 \text{ (sce)}.$$

Clearly, the coefficient of variation $c_P = \sigma(P)/E(P) = 0.41$, which is less than 1 for the exponential case.

(f) The *approximate* mean value algorithm (for general picking times) now read as follows. For $k = 1, \ldots, K$ totes, we have

$$
\begin{aligned}
E(S_e(k)) &= E(L_e(k-1))\frac{1}{\mu_e} + \frac{1}{\mu_e}, \\
E(S_c(k)) &= \frac{1}{\mu_c}, \\
E(S_i(k)) &= (E(L_i(k-1)) - \rho_i(k-1))\frac{1}{\mu_i} + \rho_i(k-1)E(R_i) + \frac{1}{\mu_i}, \quad i = 1, 2, \\
\Lambda_e(k) &= \frac{kv_e}{v_e E(S_e(k)) + 3v_c E(S_c(k)) + v_1 E(S_1(k)) + v_2 E(S_2(k))}, \\
\Lambda_c(k) &= \frac{v_c}{v_e}\ \Lambda_e(k), \\
\Lambda_i(k) &= \frac{v_i}{v_e}\ \Lambda_e(k), \quad \rho_i(k) = \Lambda_i(k)\frac{1}{\mu_i} \quad i = 1, 2, \\
E(L_e(k)) &= \Lambda_e(k)E(S_e(k)), \\
E(L_c(k)) &= \Lambda_c(k)E(S_c(k)), \\
E(L_i(k)) &= \Lambda_i(k)E(S_i(k)), \quad i = 1, 2,
\end{aligned}
$$

starting with $E(L_e(0)) = E(L_c(0)) = E(L_1(0)) = E(L_2(0)) = 0$, and where $\frac{1}{\mu_i} = E(P)$, $E(R_i) = \frac{E(P)}{2}(1+c_P^2)$. For the original (unbalanced) situation we now get the following results.

K	$\Lambda(K)$	$E(S_e(K))$	$E(S_1(K))$	$E(S_2(K))$
10	0.030	5.77	23.5	21.2
20	0.057	6.84	39.9	27.4
50	0.075	8.04	446	40.4
100	0.074	7.97	1353	39.3

Note that the performance is slightly better (as it should be, since there is less variability), but very close to that of (c). The above results are based on *approximate* mean value analysis. The approximate throughput for $K = 100$ is slightly less than the one for $K = 50$, which seems to be due to approximation error (since the exact throughput increases in K).

(g) Let x be half the width and let y be the height, such that $xy = 12 \cdot 36 = 432$. Then $X = U(0, x)$ and $Y = U(0, y)$, and

$$E(X) = \frac{1}{2}x, \quad \mathrm{var}(X) = \frac{1}{12}x^2, \quad E(Y) = \frac{1}{2}y, \quad \mathrm{var}(Y) = \frac{1}{12}y^2.$$

Hence,

$$E(P) = \frac{1}{3}(x + y) + 2, \quad \mathrm{var}(P) = \frac{1}{27}(x^2 + y^2).$$

Then it is readily verified (check!) that both $E(P)$ and $\mathrm{var}(P)$ are minimized for $x = y = \sqrt{432} = 20.8$ meter, yielding $E(P) = 15.86$ and $\mathrm{var}(P) = 32$ (so $\sigma(P) = 5.66$). For this total picking time and a balanced system, we get the following results (compare with (d)).

K	$\Lambda(K)$	$E(S_e(K))$	$E(S_1(K))$	$E(S_2(K))$
10	0.030	5.78	19.0	19.0
20	0.059	6.91	25.9	25.9
50	0.100	9.97	152	152
100	0.101	10.07	546	546